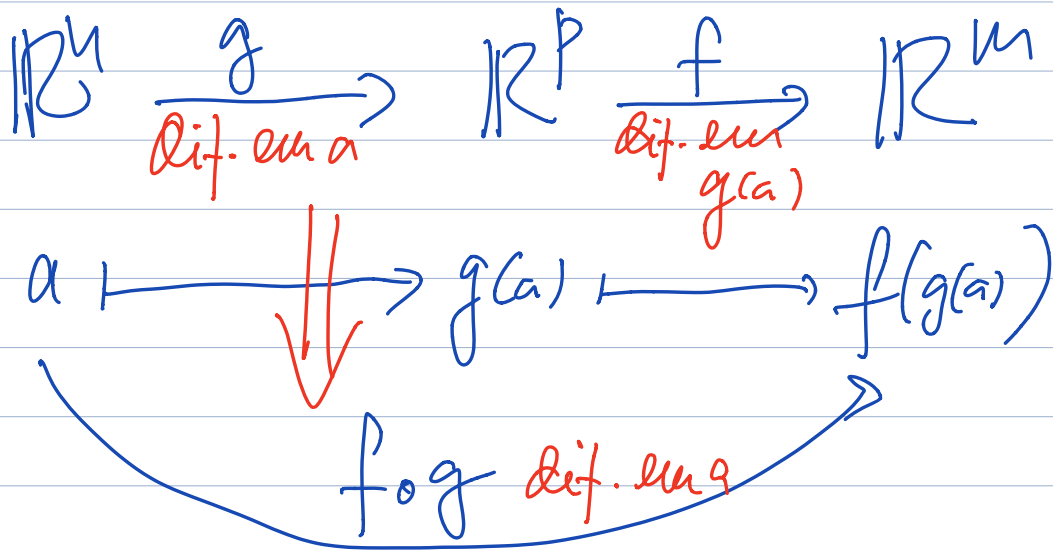


CDI-II - Prática F.4 23/3/21

Ficha 4

Teorema da função composta



$$D(f \circ g)(a) = Df(g(a)) Dg(a)$$

$n \times n$                        $m \times p$                        $p \times n$  ✓

1- Trivial ✓  $g(1,1) = (e^0, 0) = (1, 0)$

$$\begin{aligned} D(f \circ g)(1,1) &= Df(g(1,1)) Dg(1,1) \\ 3 \times 2 &= Df(1,0) Dg(1,1) \\ & \quad 3 \times 2 \quad 2 \times 2 \end{aligned}$$

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2-  $\sigma$  = "sigma",  $\gamma$  = "gamma"

$$\sigma(t) = F(\gamma(t)) = (F \circ \gamma)(t)$$

$$\begin{array}{ccccc} \textcircled{t} & \longmapsto & \textcircled{\gamma(t)} & \longmapsto & F(\gamma(t)) \\ \mathbb{R} & \longrightarrow & \mathbb{R}^3 & \longrightarrow & \mathbb{R} \end{array}$$

$$\underbrace{\hspace{10em}}_{\sigma = F \circ \gamma}$$

$$\sigma'(t) = DF(\gamma(t)) \gamma'(t)$$

$$\sigma'(t) = \nabla F(\gamma(t)) \gamma'(t)$$

$$\begin{bmatrix} \dots \end{bmatrix} \begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix} \text{matrices}$$

$$\sigma'(t) = \nabla F(\gamma(t)) \cdot \gamma'(t) \text{ vectors}$$

$$\nabla F(x, y, z) = (2x, 2y, 2z) \text{ or}$$

$$= [2x \quad 2y \quad 2z]$$

$$\gamma(t) = (\underbrace{2e^{2t}}_x, \underbrace{t^2}_y, \underbrace{\cos t}_z)$$

$$\nabla F(r(t)) = (2\sin t, 2t^2, 2\cos t)$$

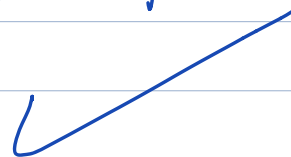
$$r'(t) = \begin{bmatrix} \cos t \\ 2t \\ -\sin t \end{bmatrix} \equiv (\cos t, 2t, -\sin t)$$

$$\sigma'(t) = (2\sin t, 2t^2, 2\cos t) \cdot (\cos t, 2t, -\sin t)$$

or

$$\begin{bmatrix} 2\sin t & 2t^2 & 2\cos t \end{bmatrix} \begin{bmatrix} \cos t \\ 2t \\ -\sin t \end{bmatrix}$$

$$\sigma'(t) = 4t^3$$



Alternativamente, usar a  
 regra da cadeia:

$$\sigma(t) = F(\gamma(t)) = F(x(t), y(t), z(t))$$

$$\left\{ \begin{array}{l} x(t) = \text{sent} \\ y(t) = t^2 \\ z(t) = \text{cost} \end{array} \right.$$

Cadeias:  $F_{xt}$ ,  $F_{yt}$ ,  $F_{zt}$

$$\sigma'_t$$

$$\sigma'(t) = \frac{\partial F}{\partial x} \frac{dx}{dt} + \frac{\partial F}{\partial y} \frac{dy}{dt} + \frac{\partial F}{\partial z} \frac{dz}{dt}$$

$$= 4t^3$$

$$\gamma(t) = (x(t), y(t), z(t))$$

(sent,  $t^2$ , cost)

$$\sigma(t) = F(x(t)) \quad \mathbb{R} \ni t \mapsto x(t) \in \mathbb{R}^3$$

$$\sigma(t) = F(x(t), y(t), z(t))$$

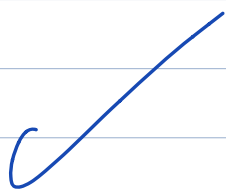
$$\sigma'(t) = \frac{d\sigma}{dt}(t) = \frac{\partial F}{\partial x} \frac{dx}{dt} + \frac{\partial F}{\partial y} \frac{dy}{dt} + \frac{\partial F}{\partial z} \frac{dz}{dt}$$

————— u —————

$$3- \quad \underbrace{D(f \circ g)(0,0)}_v = \underbrace{D(f \circ g)(0,0)}_v \quad v$$

$$D(f \circ g)(0,0) = Df(g(0,0)) Dg(0,0)$$

$$= Df(0,1,2) Dg(0,0)$$



Calculus

Calculus

$$4- \sigma(x) = f(\sin x, a + e^x)$$

$$x \xrightarrow{g} (\sin x, a + e^x) \xrightarrow{f} f(\sin x, a + e^x)$$

$$\mathbb{R} \longrightarrow \mathbb{R}^2 \xrightarrow{\quad} \mathbb{R}^3$$

$$\sigma = f \circ g$$

$$g(x) = (\sin x, a + e^x)$$

$$g: \mathbb{R} \longrightarrow \mathbb{R}^2, \quad c \neq$$

$$\sigma'(0) = Df(g(0)) Dg(0)$$

$$g(0) = (0, 1)$$

$$\sigma'(0) = Df(0,1) Dg(0)$$

$3 \times 1$                        $3 \times 2$        $2 \times 1$

$$= \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

etc... -untas ✓



5 - Regra da cadeia!

$$a) \quad \mathbb{R}^3 \xrightarrow{f} \mathbb{R}^3 \xrightarrow{g} \mathbb{R}$$

$\underbrace{\hspace{10em}}_{g \circ f}$



5-a)  $\frac{\partial}{\partial y}(g \circ f)(1,1,0)$ ?  $f(1,1,0) = (2,2,1)$

$$D(g \circ f)(1,1,0) = Dg(f(1,1,0)) \cdot Df(1,1,0)$$

$\downarrow \nabla g(2,2,1)$

$$\begin{bmatrix} \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

$$g(f(x,y,z)) = g\left(\underbrace{u(x,y,z)}_{x^2+y^2+z^2}, \underbrace{v(x,y,z)}_{x+y-z}, \underbrace{w(x,y,z)}_{xy \cdot e^z}\right)$$

$$\frac{\partial}{\partial y}(g \circ f)(x,y,z) = \frac{\partial g}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial g}{\partial v} \frac{\partial v}{\partial y} + \frac{\partial g}{\partial w} \frac{\partial w}{\partial y}$$

$(2,2,1)$        $(1,1,0)$

5-b) trivial depuis le 5-a)

$$\text{7- } \boxed{F(x, y, z) = 0}$$

$$\Leftrightarrow \boxed{z = g(x, y)}$$

$$\frac{\partial F}{\partial z}(x, y, z) \neq 0 \Rightarrow Dg(x, y)?$$

$$\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y} = ?$$

$$F(x, y, z(x, y)) = 0$$

$$h(x, y) = F(x, y, z(x, y)) \Rightarrow \bigcirc$$

type de condition :

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial z} \left[ \frac{\partial z}{\partial x} \right] = 0$$

$$\frac{\partial F}{\partial x}(x, y, z(x, y)) + \underbrace{\frac{\partial F}{\partial z}(x, y, z(x, y))}_{\neq 0} \left[ \frac{\partial z}{\partial x}(x, y) \right] = 0$$

?

$$\frac{\partial z}{\partial x}(x, y) = \frac{\partial F}{\partial x}(x, y) = - \frac{\frac{\partial F}{\partial x}(x, y, z(x, y))}{\frac{\partial F}{\partial z}(x, y, z(x, y))}$$



o e o

$$h(x, y) = m(x, y)$$

$$\frac{\partial h}{\partial x} = \frac{\partial m}{\partial x}$$

$$h(x, y) - m(x, y) = 0$$

$$\frac{\partial h}{\partial x} - \frac{\partial m}{\partial x} = 0$$

————— || —————

6- (substituted)

$$\frac{\partial}{\partial x} g \left( \underbrace{g(x^2, xy, x+y)}_{u(x,y)}, \underbrace{xy}_{v(x,y)}, \underbrace{x+y}_{w(x,y)} \right)$$

$$\rightarrow g(u(x,y), v(x,y), w(x,y))$$

$$u(x,y) = g(x^2, xy, x+y)$$

$$u(x,y) = g(a(x,y), b(x,y), c(x,y))$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial a} \frac{\partial a}{\partial x} + \frac{\partial u}{\partial b} \frac{\partial b}{\partial x} + \frac{\partial u}{\partial c} \frac{\partial c}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{\partial g}{\partial a} \frac{\partial a}{\partial x} + \frac{\partial g}{\partial b} \frac{\partial b}{\partial x} + \frac{\partial g}{\partial c} \frac{\partial c}{\partial x}$$

$$g(x,y,z) \quad g(a,b,c) \quad g(u,v,w)$$

$$g: \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$h(x,y) = g(g(x,y), y) \quad g(1,2) = 3$$

$$= g(u(x,y), v(x,y)) \quad \begin{matrix} x,y \\ (1,2) \end{matrix} \rightarrow \begin{matrix} u \ v \\ (g(1,2), 2) \end{matrix} \rightarrow g(g(1,2), 2)$$

$$(1,2) \mapsto (3,2) \rightarrow g(3,2)$$

$$\frac{\partial h}{\partial x}(1,2) = \left( \frac{\partial g}{\partial u} \right) \left( \frac{\partial u}{\partial x} \right) + \frac{\partial g}{\partial v} \frac{\partial v}{\partial x}$$

$$u(x,y) = g(x,y)$$

$$\frac{\partial u}{\partial x} = \frac{\partial g}{\partial x}$$